Algebra Problem Solving Seminar

Miklós Abért and Péter Frenkel

2013/2014 second semester, Sheet 4

1. Let
$$f(x, y) = 1 - 3x^2y^2 + x^4y^2 + x^2y^4.$$
   (a) Do we have $f \geq 0$ on $\mathbb{R}^2$?

   (b) Is f is a sum of squares in $\mathbb{R}[x, y]$?

2. In the group G, the intersection of all finite index subgroups is trivial. Does it follow that the intersection of all finite index normal subgroups is trivial?

3. The ring R with the absolute value
$$|\cdot| : R \to S,$$
   where S is a well-ordered set, is left Euclidean if for all $a, b \in R$, $b \neq 0$, there exist $q, r \in R$ such that $a = qb + r$ and $|r| < |b|$.

   (a) Is the ring $\mathbb{Z} + \mathbb{Z}i + \mathbb{Z}j + \mathbb{Z}k \subset \mathbb{H}$ with the usual absolute value left Euclidean?

   (b) Is the ring
$$\{a + bi + cj + dk | a, b, c, d \in \mathbb{Z} \text{ or } a, b, c, d \in \mathbb{Z} + 1/2\} < \mathbb{H}$$
   with the usual absolute value left Euclidean?

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